Homework 6

Some "usual computation rules" do hold for value limits $\pm\infty$ or at $\pm\infty$ ~~some~~ but not all (especially of the type $0 \cdot \infty$, e.g. $\lim_{x \to x_0} (f(x)g(x))$ when $\lim_{x \to x_0} f(x) = 0$ and $\lim_{x \to x_0} g(x) = +\infty$).

1. Let $\lim_{x \to x_0} f(x) = l > 0$ $(l \in \mathbb{R})$ and $\lim_{x \to x_0} g(x) = -\infty$. Show that $\lim_{x \to x_0} (f(x)g(x)) = -\infty$ (for simplicity let us assume throughout that functions are defined on $\mathbb{R} \setminus \{x_0\}$ in Q1, Q2, and "as large domain as it can be" in Q3, and $x_0 \in \mathbb{R}$

2. Show that $\lim_{x \to x_0} \dfrac{f(x)}{g(x)} = 0$ if $\lim_{x \to x_0} f(x) = l \in \mathbb{R}$ and $\lim_{x \to x_0} |g(x)| = +\infty$

3. Q5 of Ex 4.3 (p118 in 3rd Ed or p123 in 4th Ed of "Bartle"): Evaluate the following limits or show that they do not exist.

(a) $\lim_{x \to 1+} \dfrac{x}{x-1}$

(b) $\lim_{x \to 1-} \dfrac{x}{x-1}$

(b*) $\lim_{x \to 1} \dfrac{x}{x-1}$

(c) $\lim_{x \to 0+} \dfrac{x+2}{\sqrt{x}}$

(d) $\lim_{x \to \infty} \dfrac{x+2}{\sqrt{x}}$

(e) $\lim_{x \to 0} \dfrac{\sqrt{x+1}}{x}$ (Hint: look at side-limits)

(f) $\lim_{x \to \infty} \dfrac{\sqrt{x+1}}{x}$

(g) $\lim_{x \to +\infty} \dfrac{\sqrt{x} - 5}{\sqrt{x} + 3}$

(h) $\lim_{x \to +\infty} \dfrac{\sqrt{x} - x}{\sqrt{x} + x}$